Problem 1. Let $ABC$ be a triangle with $\angle BAC \neq 90^\circ$. Let $O$ be the circumcenter of the triangle $ABC$ and let $\Gamma$ be the circumcircle of the triangle $BOC$. Suppose that $\Gamma$ intersects the line segment $AB$ at $P$ different from $B$, and the line segment $AC$ at $Q$ different from $C$. Let $ON$ be a diameter of the circle $\Gamma$. Prove that the quadrilateral $APNQ$ is a parallelogram.

Problem 2. For a positive integer $k$, call an integer a pure $k$-th power if it can be represented as $m^k$ for some integer $m$. Show that for every positive integer $n$ there exist $n$ distinct positive integers such that their sum is a pure 2009-th power, and their product is a pure 2010-th power.

Problem 3. Let $n$ be a positive integer. $n$ people take part in a certain party. For any pair of the participants, either the two are acquainted with each other or they are not. What is the maximum possible number of the pairs for which the two are not acquainted but have a common acquaintance among the participants?

Problem 4. Let $ABC$ be an acute triangle satisfying the condition $AB > BC$ and $AC > BC$. Denote by $O$ and $H$ the circumcenter and the orthocenter, respectively, of the triangle $ABC$. Suppose that the circumcircle of the triangle $AHC$ intersects the line $AB$ at $M$ different from $A$, and that the circumcircle of the triangle $AHB$ intersects the line $AC$ at $N$ different from $A$. Prove that the circumcenter of the triangle $MNH$ lies on the line $OH$.

Problem 5. Find all functions $f$ from the set $\mathbb{R}$ of real numbers into $\mathbb{R}$ which satisfy for all $x, y, z \in \mathbb{R}$ the identity

$$f(f(x) + f(y) + f(z)) = f(f(x) - f(y)) + f(2xy + f(z)) + 2f(xz - yz).$$