Problem 1. We call a 5-tuple of integers arrangeable if its elements can be labeled \(a, b, c, d, e\) in some order so that \(a - b + c - d + e = 29\). Determine all 2017-tuples \(n_1, n_2, \ldots, n_{2017}\) such that if we place them in a circle in clockwise order, then any 5-tuple of numbers in consecutive positions on the circle is arrangeable.

Proposed by Warut Suksompong, Thailand

Problem 2. Let \(ABC\) be a triangle with \(AB < AC\). Let \(D\) be the intersection point of the internal bisector of angle \(BAC\) and the circumcircle of \(ABC\). Let \(Z\) be the intersection point of the perpendicular bisector of \(AC\) with the external bisector of angle \(\angle BAC\). Prove that the midpoint of the segment \(AB\) lies on the circumcircle of triangle \(ADZ\).

Proposed by Equipo Nicaragua, Nicaragua

Problem 3. Let \(A(n)\) denote the number of sequences \(a_1 \geq a_2 \geq \ldots \geq a_k\) of positive integers for which \(a_1 + \cdots + a_k = n\) and each \(a_i + 1\) is a power of two \((i = 1, 2, \ldots, k)\). Let \(B(n)\) denote the number of sequences \(b_1 \geq b_2 \geq \ldots \geq b_m\) of positive integers for which \(b_1 + \cdots + b_m = n\) and each inequality \(b_j \geq 2b_{j+1}\) holds \((j = 1, 2, \ldots, m - 1)\).

Prove that \(A(n) = B(n)\) for every positive integer \(n\).

Proposed by Senior Problems Committee of the Australian Mathematical Olympiad Committee
Problem 4. Call a rational number $r$ powerful if $r$ can be expressed in the form $\frac{p^k}{q}$ for some relatively prime positive integers $p, q$ and some integer $k > 1$. Let $a, b, c$ be positive rational numbers such that $abc = 1$. Suppose there exist positive integers $x, y, z$ such that $a^x + b^y + c^z$ is an integer. Prove that $a, b, c$ are all powerful.

Proposed by Jeck Lim, Singapur

Problem 5. Let $n$ be a positive integer. A pair of $n$-tuples $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ with integer entries is called an exquisite pair if

$$|a_1b_1 + \cdots + a_nb_n| \leq 1.$$  

Determine the maximum number of distinct $n$-tuples with integer entries such that any two of them form an exquisite pair.

Proposed by Pakawut Jiradilok and Warut Suksompong, Thailand